Uncertainty Specification and Propagation for Loss Estimation Using FOSM Methods

J.W. Baker and C.A. Cornell
Dept. of Civil and Environmental Engineering, Stanford University, Stanford, CA 94305-4020

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ABSTRACT: Probabilistic prediction of structural and nonstructural damage costs due to future earthquakes is one component of loss estimation currently being developed for use in performance-based earthquake engineering. Sources of uncertainty in this prediction include epistemic and aleatory uncertainty in the site ground motion hazard, the building response, the damage measures of each of the many building elements, and repair cost of each of the elements. These are inter- and cross-correlated random variables. Two desired results are the total uncertainty in annual losses, and the contribution of each uncertainty source to the total uncertainty. Monte Carlo simulation is a simple solution, but it can be computationally expensive. This study proposes an alternative approach using First-Order Second-Moment (FOSM) methods for all but the (dominant) ground motion intensity variable. Suggestions for characterization of correlations are presented. A procedure for applying FOSM methods in the calculation of total uncertainty is outlined. The proposed technique is very efficient, and easily used for sensitivity studies.

1 INTRODUCTION

Estimation of annual losses in a building due to earthquake damage is a quantity of interest to decision makers, and is a current topic of study in performance-based earthquake engineering. Among the quantities to be determined are the uncertainty in the result, and the contribution of each source of uncertainty to the total uncertainty. The Pacific Earthquake Engineering Research Center (PEER) has proposed the following framing equation for this analysis:

$$\lambda_{TC}(z) = \int\limits_{u}\int\limits_{v}\int\limits_{y}\int\limits_{x} f_{TC|\mathbf{DVE}}(z,\mathbf{u}) f_{\mathbf{DVE}|\mathbf{DM}}(\mathbf{u},\mathbf{v}) f_{\mathbf{DM}|\mathbf{EDP}}(\mathbf{v},\mathbf{y}) f_{\mathbf{EDP}|IM}(\mathbf{y},x) \left|d\lambda_{IM}(x)\right| \tag{1}$$

with terms defined in Section 1.1. This equation allows for modular consideration of the ground motion hazard, building response, damage to building elements, element repair costs, and total repair cost (Cornell and Krawinkler, 2000, Porter 2001, Krawinkler, 2002).

One option for calculating uncertainty in the result is through Monte Carlo simulation (Porter, 2001). Simulation methods have straightforward methodology and quantifiable accuracy, but can be expensive computationally, especially when multiple runs are required to calculate sensitivities. The objective of this study is to propose an alternative method of calculating uncertainty using the First-Order Second-Moment (FOSM) method (e.g. Melchers, 1999). We shall use this approximate method to "collapse out" several of the intermediate conditional random variables, leaving a mean and variance of Total Cost (TC) conditioned on the Intensity Measure (IM) of the ground motion. This information can then be combined (using numerical integration) with the ground motion hazard, $|d\lambda_{IM}(x)|$, to obtain the expected annual total cost, variance in annual total cost, and the annual rate of exceeding a given total cost.

1.1 *Explanation of the Framing Equation*

The variables in Equation 1 are defined as follows:
$\lambda_{TC}(z)$ is the annual rate of exceeding a total repair cost of z, where total repair cost, TC is the decision variable under study.
$f_{TC|\mathbf{DVE}}(z,\mathbf{u})$ is the PDF of TC, conditioned on the vector of damage values of each element (DVE_j is

the damage value of element j). The assumption in the framework described below is that the total cost of repair is the sum of all element repair costs, but this can be easily generalized.

$f_{\mathbf{DVE}|\mathbf{DM}}(\mathbf{u},\mathbf{v})$ is the PDF of the vector of damage values of each element, given the vector of damage states of each element (DM_j is the damage state of element j). Mean repair costs can be estimated from sources such as R.S. Means Co.'s published materials on construction cost estimating (R.S. Means Co. 2002). Additional quantification of repair costs is a topic of current research.

$f_{\mathbf{DM}|\mathbf{EDP}}(\mathbf{v},\mathbf{y})$ is the PDF of the vector of damage states, given the vector of engineering demand parameters. In current research, these damage states are typically discrete, and each state is described by a fragility function, which returns the probability of an element exceeding the damage state at a given *EDP* level. See Aslani and Miranda (2002) for examples.

$f_{\mathbf{EDP}|IM}(\mathbf{y},x)$ is the PDF of the vector of engineering demand parameters, given the intensity measure. For aleatory uncertainty, this distribution can be determined using, for example, Incremental Dynamic Analysis (Vamvatsikos and Cornell 2002).

$|d\lambda_{IM}(x)|$ is the absolute value of the derivative of the annual rate of exceeding a given value of the intensity measure (the seismic hazard curve). The absolute value is needed because the derivative is negative. See, for example, Kramer (1995) for background on hazard curves.

2 ASSUMPTIONS

A Markovian dependence is assumed for all distributions in the framework. For example, it is assumed that the distribution of the DM vector can be conditioned solely on the EDP vector, and that knowledge of the IM provides no additional information. In this way, previous conditioning information does not need to be carried forward through all future distributions, reducing complexity. A conditioning variable that contains all necessary conditional information is deemed a "sufficient" descriptor (Luco 2002).

All damage is assumed to occur on an element level. The total cost of damage to the structure is then the sum of the damage cost of each element in the structure. The exception to this assumption is when collapse occurs, and repair costs will be a function of the collapse, rather than individual element responses. The treatment of this exception is explained below.

All relations in the framework are assumed to be scalar functions. For example, the conditional distribution of the Damage Measure of element j is a function of only the ith Engineering Demand Parameter. Or alternatively,

$$f_{DM_j|\mathbf{EDP}}(v_j,\mathbf{y}) = f_{DM_j|EDP_i}(v_j,y_i) \tag{2}$$

Note also that the function is not conditioned on variables from any previous steps, because of the Markovian process assumption described earlier.

To calculate total uncertainty in our decision variable, it will be necessary to account for both epistemic and aleatory uncertainty. The framework outlined here is appropriate for either source of uncertainty. These two sources of uncertainty are uncorrelated, allowing their contributions to be calculated separately for simplicity, and towards the end of the procedure. This is further discussed in Section 3.6.

These assumptions are believed to be consistent with the most advanced current seismic loss estimation efforts. Most can be relaxed without formal difficulty.

3 PROCEDURE

The procedure outlined makes use of FOSM approximations to calculate the mean and variance of *TC* given *IM*. This information can then be combined with the ground motion hazard, $|d\lambda_{IM}(x)|$, to obtain the expected annual total cost, variance in annual total cost, and (together with a distribution type assumption), the mean annual rate of exceeding a given total cost. For this final combination with the ground motion hazard, FOSM approximations are not used. The FOSM approximations are justified by the assumption that the uncertainty in the *IM* hazard curve is the most significant contributor to variance of the total loss. Therefore, we are retaining the full distribution for *IM* itself, but using the FOSM approximations for all moments conditioned on *IM*. In addition, we likely do not have information about the full distributions of some variables (for example, repair costs), and so neglecting higher moments of these distributions does not result in a significant loss of available information.

Note that we are working with natural logarithms of the variables described previously. This allows us to work with sums of terms, rather than products. We revert to a non-log form for the final result. The procedure is outlined in the following sections.

3.1 *Specify* $\ln \mathbf{EDP} \mid IM$

The proposed model in this study is $EDP_i|IM=H_i(IM)\varepsilon_i(IM)$, where $H_i(IM)$ is the (deterministic) mean value of EDP_i given IM, and $\varepsilon_i(IM)$

is a random variable with mean of one, and conditional variance adjusted to model the variance in EDP_i. (We introduce the random variable notation $X|Y$, to denote that the model of X is conditioned on Y.) Then when we use the log form of EDP_i, we have a random variable of the form $\ln(EDP_i|IM)=\ln(H_i(IM))+\ln(\varepsilon_i(IM))$. Note that the expected value of $\ln(EDP_i|IM)$ is $\ln(H_i(IM))$, and the variance of $\ln(EDP_i|IM)$ is equal to the variance of $\ln(\varepsilon_i(IM))$. Both $\ln(H_i(IM))$ and $\mathrm{Var}[\ln(\varepsilon_i(IM))]$, as well as the correlations between $\ln EDP$'s, can be determined from Incremental Dynamic Analysis. We will need the following information for our calculations:

$$E[\ln EDP_i \mid IM], \text{ denoted } h_i(IM) \text{ for all } EDP_i \tag{3}$$

$$Var[\ln EDP_i \mid IM], \text{ denoted } h^*_i(IM) \text{ for all } EDP_i \tag{4}$$

$$\rho(\ln EDP_i, \ln EDP_j \mid IM), \text{ denoted } \hat{h}_{ij}(IM) \text{ for all } \{EDP_i, EDP_j\} \tag{5}$$

These functions will be used in Section 3.3 below.

3.2 *Specify* $\mathbf{DM} \mid \ln \mathbf{EDP}$ *and* $\ln \mathbf{DVE} \mid \mathbf{DM}$, *and collapse to* $\ln \mathbf{DVE} \mid \ln \mathbf{EDP}$

The discrete states of the Damage Measure variable found in current loss estimation (Aslani and Miranda, 2002, Porter, 2001) are not compatible with the FOSM approach, which requires continuous functions for the moments. To deal with the discrete states, we take advantage of the fact that we can always "collapse" the two distributions $\mathbf{DM} \mid \ln \mathbf{EDP}$ and $\ln \mathbf{DVE} \mid \mathbf{DM}$ into one continuous distribution $\ln \mathbf{DVE} \mid \ln \mathbf{EDP}$ by integrating over the appropriate variable:

$$f_{\mathbf{DVE|EDP}}(\mathbf{u},\mathbf{y}) = \int_{\mathbf{v}} f_{\mathbf{DVE|DM}}(\mathbf{u},\mathbf{v}) f_{\mathbf{DM|EDP}}(\mathbf{v},\mathbf{y}) \tag{6}$$

For a given element with n possible damage states, we use a set of element fragility functions $F_1, F_2 \ldots F_n$, such that $F_i(y)=P(DM>d_i|EDP=y)$ (see Figure 1). We also define $F_0\equiv 1$ (the probability that each element has at least zero damage is one). These functions will have a corresponding set of distributions $c_1, c_2 \ldots c_n$ of element repair costs such that $c_i(v)$ is a probability distribution of DVE, given that the damage state equals d_i (see Figure 2). With this information, we can determine the first two moments of the collapsed distributions. For example, Aslani and Miranda (2002) document the development of one set of these functions.

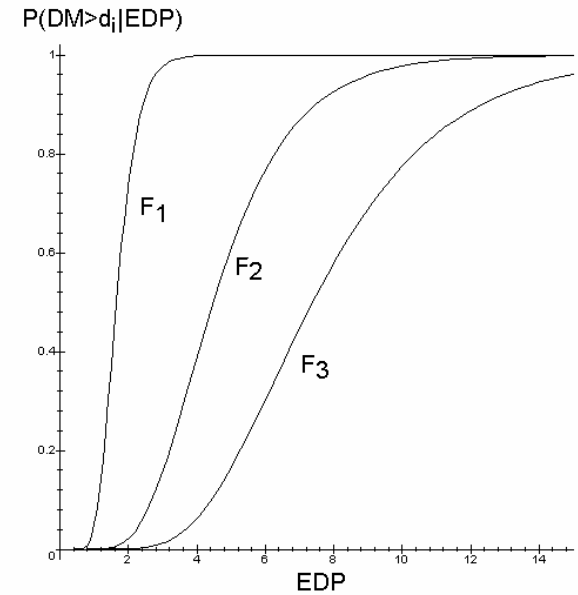

Figure 1: Example Element Fragility Functions

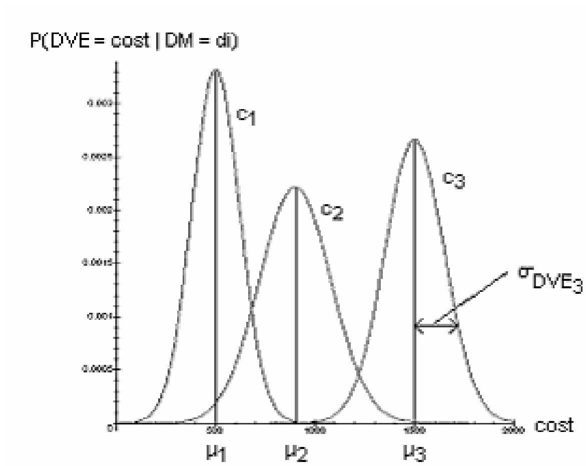

Figure 2: Element Repair Costs

From the total probability theorem, we know that in this case, Equation 6 can be written in scalar form for each DVE as:

$$f_{DVE|EDP} = \sum_{\text{Damage States}} f_{DVE|DM=d_i} P_{DM=d_i|EDP} \tag{7}$$

(recall our assumption that each DVE is dependent on a single EDP). For our FOSM purposes, furthermore, it is sufficient to find simply the conditional means, variances, and covariances of the DVE's given the EDP's. Thus, taking the mean of this PDF, we have the result:

$$E[DVE \mid EDP] = \sum_{\substack{Damage\\States}} \mu_i \left(F_i(EDP) - F_{i+1}(EDP)\right) \tag{8}$$

Applying the same thinking to $E[DVE^2 \mid EDP]$, and recognizing that $Var[X]=E[X^2]-\mu_X^2$, we have the following result:

$$Var[DVE \mid EDP] = \sum_{\substack{Damage\\States}} \sigma^2_{DVE_i} ((F_i(EDP) - F_{i+1}(EDP)) \tag{9}$$
$$+ \sum_{\substack{Damage\\States}} (\mu_i - \overline{\mu})^2 (F_i(EDP) - F_{i+1}(EDP))$$

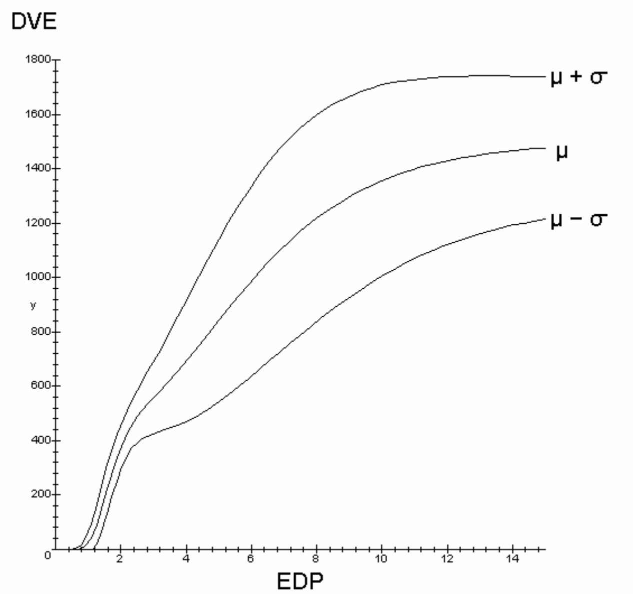

Figure 3: Collapsed distribution *DVE* | *EDP*

Figure 3 shows an example of the mean and mean plus or minus one sigma, as generated from the example distributions shown in Figure 1 and Figure 2.

3.2.1 *Quantifying Correlations*

We now need to determine correlations among the *DVE*'s of all elements in the structure. Note that like the mean and variance, these correlations are conditioned on the *EDP*'s. While these calculations are straightforward, estimation of the necessary correlation inputs is a difficult task due to a lack of data. In the absence of additional information, it may be helpful to use the following characterization scheme. Let us assume for this purpose a model of the form:

$$\ln DVE_k \mid \ln EDP_i = g_k(\ln EDP_i) + \ln \varepsilon_{Struc} + \ln \varepsilon_{ElClass_m} + \ln \varepsilon_{El_k} \tag{10}$$

where ε_{Struc} represents uncertainty common to the entire structure, $\varepsilon_{ElClass_m}$ represents uncertainty common only to elements of class "*m*" (e.g. drywall partitions, moment connections, etc.), and ε_{El_k} represents uncertainty unique to element *k*. All of these ε's are assumed to be mutually uncorrelated. We then define:

$$Var[\ln \varepsilon_{Struc} \mid \ln EDP_i] = \beta_{Struc}{}^2 \tag{11}$$

$$Var[\ln \varepsilon_{ElClass_m} \mid \ln EDP_i] = \beta_{ElClass}{}^2 \text{ for all m} \tag{12}$$

$$Var[\ln \varepsilon_{El_k} \mid \ln EDP_i] = \beta_{El}{}^2 \text{ for all } k \tag{13}$$

Then the variance of $\ln DVE_k | \ln EDP_i$ is the sum of these variances. For this special case, a simple closed form solution exists for the correlation coefficient between two element *DVE*'s. If the two elements are of the same element class, then:

$$\rho(\ln DVE_k, \ln DVE_l \mid \ln EDP_i, \ln EDP_j) = \frac{\beta_{Struc}{}^2 + \beta_{ElClass}{}^2}{\beta_{Struc}{}^2 + \beta_{ElClass}{}^2 + \beta_{El}{}^2} \tag{14}$$

If the elements are of different element classes, then their correlation coefficient is given by:

$$\rho(\ln DVE_k, \ln DVE_l \mid \ln EDP_i, \ln EDP_j) = \frac{\beta_{Struc}{}^2}{\beta_{Struc}{}^2 + \beta_{ElClass}{}^2 + \beta_{El}{}^2} \tag{15}$$

Note that this formulation requires $\beta_{ElClass}{}^2$ to be equal for all element classes, and $\beta_{El}{}^2$ to be equal for all elements. If this is excessively limiting, a closed form solution also exists that allows $\beta_{ElClass}{}^2$ to vary by class, and $\beta_{El}{}^2$ to vary by element (Baker and Cornell, 2002). The principles used for this solution are developed in Ditlevsen (1981).
The use of more than two uncertain terms, and the use of β^2 terms that vary by class or element are both generalizations of the basic equicorrelated model. Thus, we will refer to a model incorporating any of these generalizations as a *generalized equicorrelated model*. The correlation matrix for a generalized equicorrelated model will have off-diagonal terms that vary from term to term, as opposed to the strict equicorrelated model, where all off-diagonal terms are identical.

We have now concluded the collapse of the distribution $\ln \mathbf{DVE} | \ln \mathbf{EDP}$. We have the conditional mean and variance functions of $\ln DVE_k | \ln EDP_i$, obtained by collapsing the distributions provided (see Equations 8 and 9), and correlation coefficients determined using the generalized equicorrelated model (see Equations 14 and 15). We choose for future notational clarity to denote these results as:

$E[\ln DVE_k \mid \ln EDP_i]$, denoted $g_k(\ln EDP_i)$ for all DVE_k (16)

$Var[\ln DVE_k \mid \ln EDP_i]$, denoted $g^*_k(\ln EDP_i)$ for all DVE_k (17)

$\rho(\ln DVE_k, \ln DVE_l \mid \ln EDP_i, \ln EDP_j)$, denoted $\hat{g}_{kl}(\ln EDP_i, \ln EDP_j)$ for all $\{DVE_k, DVE_l\}$ (18)

With this information quantified, we can now use it along with the results from Section 3.1 to calculate $\ln \mathbf{DVE} | IM$.

3.3 *Calculate* $\ln \mathbf{DVE} \mid IM$

Using information from above, we can calculate the first and second moments of $\ln \mathbf{DVE} \mid IM$. This involves collapsing out the dependence on $\mathbf{EDP}$, as suggested in Equation 19 below.

$$f_{\mathbf{DVE}|IM}(\mathbf{u},x)=\int_{\mathbf{y}} f_{\mathbf{DVE}|\mathbf{EDP}}(\mathbf{u},\mathbf{y})\, f_{\mathbf{EDP}|IM}(\mathbf{y},x) \tag{19}$$

To maintain tractability, we shall use an FOSM approximation here. To remove dependence on $\mathbf{EDP}$, we take the expectation of $\ln \mathbf{DVE}$ of with respect to $\ln \mathbf{EDP}$ (given IM). We write this as $E[\ln DVE_k|IM]=E_{EDPi|IM}[E[\ln DVE_k|\ln EDP_i]]$, where $E_{EDPi|IM}[\cdot]$ denotes this particular conditional expectation operator. Substituting our notation from Equations 16 and 17, we have:

$$E[\ln DVE_k \mid IM] \cong g_k\left(h_i(IM)\right) \tag{20}$$

Using a similar approach to conditional moments, and using the result from probability theory:

$$Var[X]=E\left[Var[X \mid Z]\right]+Var\left[E[X \mid Z]\right] \tag{21}$$

we can also derive the variance and covariances of $\ln DVE_k \mid IM$ with the usual FOSM approximations:

$$Var[\ln DVE_k \mid IM] \cong g^*_k\,(h_i(IM))+\left(\frac{\partial g_k}{\partial \ln EDP_i}\right)^2 \Bigg|_{h_i(IM)} h^*_i\,(IM) \tag{22}$$

$$\begin{aligned} Cov[\ln DVE_k, DVE_l \mid IM] \cong\; & \hat{g}_{kl}(h_i(IM),h_j(IM))\sqrt{g^*_k\,(h_i(IM))}\sqrt{g^*_l\,(h_j(IM))} \\ & +\left(\frac{\partial g_k}{\partial \ln EDP_i}\right)\Bigg|_{h_i(IM)}\left(\frac{\partial g_l}{\partial \ln EDP_j}\right)\Bigg|_{h_j(IM)} \hat{h}_{ij}(IM)\sqrt{h^*_i\,(IM)}\sqrt{h^*_j\,(IM)} \end{aligned} \tag{23}$$

3.4 *Switch to the non-log form* $\mathbf{DVE} \mid IM$

To switch to the non-log form of DVE, we can use the first-order approximation $E[e^X] \simeq e^{E[X]}$. Then we have the following results:

$$E[DVE_k \mid IM] \cong e^{g_k(h_i(IM))} \tag{24}$$

$$Var[DVE_k \mid IM] \cong e^{2g_k(h_i(IM))} Var[\ln DVE_k \mid IM] \tag{25}$$

$$Cov[DVE_k, DVE_j \mid IM] = e^{g_k(h_i(IM))+g_l(h_j(IM))} Cov[\ln DVE_k, \ln DVE_l \mid IM] \tag{26}$$

These element results can now be used to compute the moments of $TC \mid IM$.

3.5 *Compute moments of* $TC \mid IM$

Under the assumption that Total Cost is the sum of element costs, we can now aggregate the results from all individual elements to compute an expectation and variance for the total cost of damage to the entire building. The expected total cost is the sum of expected element costs:

$$E[TC \mid IM]=\sum_{k=1}^{\#elements} E[DVE_k \mid IM] \tag{27}$$

We denote the expected value computed in Equation 27 as $q(IM)$. The variance of total cost can is the sum of element variances, including covariances between element costs:

$$\begin{aligned} Var[TC \mid IM] \;=\; & \sum_{k=1}^{\#elements} Var[DVE_k \mid IM] \\ & +2\sum_{k=1}^{\#elements}\sum_{l=k+1}^{\#elements} Cov[DVE_k, DVE_l \mid IM] \end{aligned} \tag{28}$$

We denote the expected value computed in Equation 28 as $q^*(IM)$.

3.6 *Repeat Procedure to Calculate Epistemic Uncertainty*

We assume a model of the form $TC|IM=q(IM)\,\varepsilon_R\varepsilon_U$, where q(IM) is the best estimate of the conditional mean as calculated in Equation 27, and ε_R and ε_U are uncorrelated random variables representing aleatory and epistemic uncertainty, respectively. Then

$$\ln TC \mid IM = \ln q(IM)+\ln\varepsilon_R+\ln\varepsilon_U \tag{29}$$

Because ε_R and ε_U are uncorrelated, we may deal with them in separate steps. The above procedure using aleatory uncertainty alone allowed us to find the variance due to ε_R. We must now repeat the procedure to calculate the variance due to ε_U. The total uncertainty can then be calculated by combining the two uncertainties as follows:

$$Var[\ln TC \mid IM]=Var[\ln\varepsilon_R]+Var[\ln\varepsilon_U] \tag{30}$$

We denote this $\beta_{TC|IM}{}^2$. Note that we have switched to logs again to allow use of sums rather than products. The change can be made using the following relationship:

$$Var[\ln\varepsilon_R] \cong \ln\left(1+\frac{Var_R[TC \mid IM]}{E_R[TC \mid IM]^2}\right) \tag{31}$$

We denote $Var[\ln \varepsilon_R]$ as $\beta_R{}^2$. From this value, we can then calculate $Var[\ln TC \mid IM]$ using Equation 30. Once we have calculated $Var[\ln TC \mid IM]$, accounting for both aleatory and epistemic uncertainty, we can denote this value as $q^*(IM)$, and use it in the equations to follow.

When repeating the procedure to find the variance in TC given IM due to epistemic uncertainty, epistemic uncertainties for each conditional random variable will need to be estimated, and characterization of correlation is again a challenging task. It is suggested that the generalized equicorrelated model developed in Section 3.2.1 may be used effectively for this problem.

4 ACCOUNTING FOR COLLAPSE CASES

At high *IM* levels, the potential exists for a structure to experience collapse (defined here as extreme deflections at one or more story levels). In this building state, repair costs are more likely a function of the collapse, rather than individual element damage. In fact, the structure is likely not to be repaired at all. Thus, our predicted loss may not be accurate in these cases. In addition, the large deflections predicted in a few cases will skew our expected values of some *EDP*'s such as interstory drifts, although collapse is only occurring in a fraction of cases. To account for the possibility of collapse, we would like to use the technique outlined above for no-collapse cases, and allow for an alternate loss estimate when collapse occurs. The following modification is suggested. Note, in the following calculations, we are conditioning on a collapse indicator variable. To communicate this, we have denoted the collapse and no collapse condition as "*C*" and "*NC*" respectively.

- At each IM level, compute the probability of no collapse. This probability, $p(NC \mid IM)$, is simply the fraction of analysis runs where no collapse occurs
- Calculate results using the FOSM analysis as before, but using only the runs that resulted in no collapse. We now denote these results $E[TC \mid IM, NC]$ and $Var[TC \mid IM, NC]$.
- Define an expected value and variance of total cost, given that collapse has occurred, denoted $E[TC \mid IM, C]$ and $Var[TC \mid IM, C]$. These values will likely not be functions of *IM*, but the conditioning on *IM* is still noted for consistency.

The expected value of *TC* for a given *IM* level is now the average of the collapse and no collapse *TC*, weighted by their respective probabilities of occurring:

$$E[TC \mid IM] = p(NC \mid IM)E[TC \mid IM, NC] + (1 - p(NC \mid IM))E[TC \mid IM, C] \tag{32}$$

The variance can be computed using the property from Equation 21:

$$Var[TC \mid IM] = \begin{bmatrix} p(NC \mid IM)Var[TC \mid IM, NC] \\ +(1 - p(NC \mid IM))Var[TC \mid IM, C] \end{bmatrix} + \begin{bmatrix} p(NC \mid IM)(E[TC \mid IM] - E[TC \mid IM, NC])^2 \\ +(1 - p(NC \mid IM))(E[TC \mid IM] - E[TC \mid IM, C])^2 \end{bmatrix} \tag{33}$$

The procedure can now be implemented as before, using these moments. This collapse-case modification is probably necessary for any implementation of the model, as analysis of shaking (*IM*) levels sufficient to cause large financial loss are likely also to cause collapse in some representative ground motion records.

5 INCORPORATE THE SITE HAZARD

The expected value and variance of *TC* given *IM* can now be incorporated with the site hazard to compute the expected annual loss, and the rate of exceeding a given Total Cost.

5.1 *Annual Loss*

Using the functions $q(IM)$ and $q^*(IM)$, and the derivative of the hazard curve, $d\lambda(IM)$, the mean and variance of *TC* per annum can be calculated by numerical integration:

$$E[TC] = \int_{IM} q(IM) \left| d\lambda(IM) \right| \tag{34}$$

$$\begin{aligned} Var[TC] &= E[Var[TC \mid IM] + Var[E[TC \mid IM]] \\ &= \int_{IM} q^*(IM)\left|d\lambda(IM)\right| + \int_{IM} q^2(IM)\left|d\lambda(IM)\right| - E[TC]^2 \end{aligned} \tag{35}$$

Note that the first term of Equation 35 is the contribution from uncertainty in the cost function given *IM*, and the second two terms are the contribution from uncertainty in the *IM*.

5.2 *Rate of Exceedance of a Given TC*

The first and second moment information for *TC*|*IM* can be combined with a site hazard to compute $\lambda_{TC}(z)$, the annual frequency of exceeding a *given* Total Cost z. For this calculation, it is necessary to assume a probability distribution for *TC*|*IM* that has conditional mean and variance equal to the values calculated previously. The rate of exceedance of a given *TC* is then given by:

$$\lambda_{TC}(z) = \int_{IM} F_{TC|IM}(z, x) \left| d\lambda_{IM}(x) \right| \tag{36}$$

5.3 *Analytic Solution*

Generally, the integral above will require a numerical integration. However, if the following simplifying assumptions are made, an analytic solution is available:

- The distribution of *TC*|*IM* is lognormal
- $\beta_{TC|IM}$ is approximated as constant for all *IM*; we call this constant value $\beta^*_{TC|IM}$
- E[*TC*|*IM*] is approximated by a function of the form $a'IM^b$, where a and b are constants. Note that this is consistent with fitting the *median* of *TC*|*IM* with aIM^b, where

$$a = a' e^{-\frac{1}{2}\beta^{*2}_{TC|IM}} \tag{37}$$

- An approximate hazard curve of the form $\lambda_{IM}(x)=k_0x^{-k}$ is fit to the true site hazard curve

Under these conditions, the annual rate of exceeding a given Total Cost is given by:

$$\lambda_{TC}(z) = k_0\left(\frac{z}{a'}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\beta^*_{TC|IM}{}^2\right) \qquad (38)$$

We note that if the a from Equation 37 is substituted into Equation 38, then the result becomes

$$\lambda_{TC}(z) = k_0\left(\frac{z}{a}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k^2}{b^2}\beta^*_{TC|IM}{}^2\right) \qquad (39)$$

This equation is useful as an efficient estimate of $\lambda_{TC}(z)$, but it is also very informative as a measure of the relative importance of uncertainty in the calculation. The term:

$$k_0\left(\frac{z}{a'}\right)^{-k/b} \qquad (40)$$

in the Equation 38 would be the result if $\beta^*_{TC|IM}$ were to equal zero – that is if we made all calculations only using expected values and neglected cost uncertainty given IM. The term:

$$\exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\beta^*_{TC|IM}{}^2\right) \qquad (41)$$

in Equation 38 is an amplification factor that varies with the uncertainty in $TC|IM$ present in the problem. Thus for this special case, it is simple to calculate the effect of uncertainty on the rate of exceeding a given Total Cost. As we shall show below, it may not be unreasonable for this factor to increase $\lambda_{TC}(z)$ by a factor of 10, so the effect of uncertainty may very well be significant. However, even for large values of $\beta^*_{TC|IM}$, the annual rate of exceedance is still dominated by the term from Equation 40. It is for this reason that it has been proposed here that the FOSM approximations of $\beta^*_{TC|IM}$ performed above are sufficient to provide an accurate result.
For illustration, let us assume that the expected TC as a function of IM has been estimated using the above technique as:

$$E[TC \mid IM] = 1 - e^{-2IM^2} \qquad (42)$$

This function could be approximated by the function:

$$E[TC \mid IM] = 1.4IM^{1.8} \qquad (43)$$

A plot of these two functions is shown in Figure 4. Note that the analytic function is a good fit over the range $0<TC<0.5$.

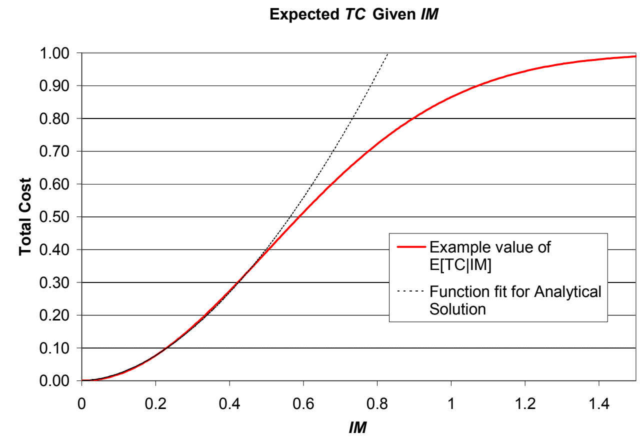

Figure 4: Expected $TC \mid IM$, for $E[TC \mid IM] = 1 - e^{-2IM^2}$ and $1.4IM^{1.8}$ fit

5.4 *Comparison of Results from Numerical Integration and Analytic Solution*

Both the numerical solution and the analytical solution outlined above in Sections 5.2 and 5.3, respectively, can be evaluated for several values of TC. The results can then be plotted to generate a loss curve, as shown below in Figure 5. This figure was generated using the expected TC curves shown in Figure 4 above. For both solutions, we have assumed a hazard curve of the form:

$$\lambda_{IM}(x) = k_0 x^{-k} \qquad (44)$$

where k_0 and k are constants equal to 0.002 and 3, respectively. We have also assumed $\beta_{TC|IM}$ equal to 0.6 for both solutions.

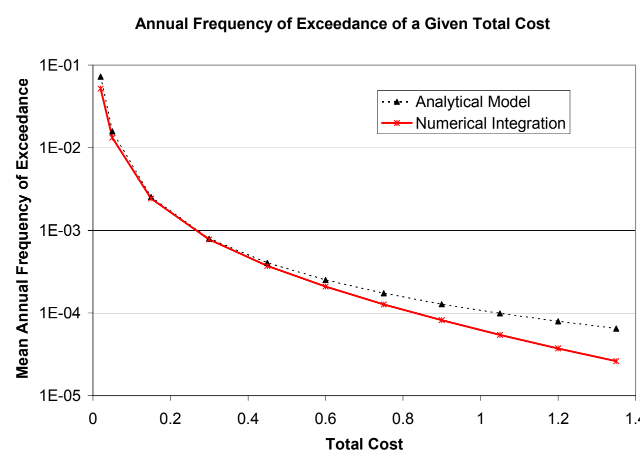

Figure 5: $\lambda_{TC}(z)$: Comparison of numerical integration and analytic solution

Figure 5 also allows us to compare the analytical solution to the numerical one. For the functions given in Equations 42 and 43, the analytic solution is a good approximation of the numeric solution over the TC range where Equation 43 closely fit Equation 42 ($0<TC<0.5$). As we move to higher TC levels, where the analytical solution was not a good fit, the $\lambda_{TC}(z)$ results also diverge.

6 EFFECT OF UNCERTAINTY ON LOSSES

The variance in annual losses is the result that most explicitly shows the effects of uncertainty. However, uncertainty also has an effect on the annual rate of exceeding a given *TC*. This is most clearly seen in the analytical solution of Equation 38, where $\beta^*_{TC|IM}$ appears in the equation for $\lambda_{TC}(z)$. This is discussed in Section 5.3. The anticipated effect is for uncertainty to increase the rate of occurrence of a given *TC*. However, depending on the slopes of the hazard curve and mean of *TC* as a function of *IM*, (defined by the parameters *k* and *b*), increasing $\beta^*_{TC|IM}$ can potentially decrease $\lambda_{TC}(z)$, or have no effect at all. Using the more general numerical integration of Equation 36, we find similar results, as shown in Figure 6. In this example, using the functions assumed previously, we see that the shift in results is minor for $\beta^*_{TC|IM}<1$, but for $\beta^*_{TC|IM}=2$, the expected annual frequency of occurrence of large costs has increased by approximately an order of magnitude. Because our expected repair cost function does not ever produce a loss greater than 1, the inclusion of uncertainty is critical for estimating occurrence of total costs greater than 1, as seen in the figure.

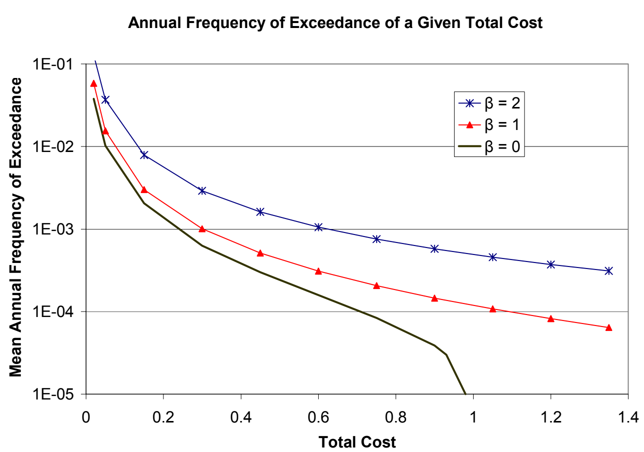

Figure 6: Effect of Uncertainty on Frequency of Exceedance of Total Cost

The important conclusion to be drawn from this result is that using expected values alone and ignoring uncertainties, although tempting because of its ease, can potentially lead to inaccurate results.

CONCLUSION

A procedure for estimation of uncertainty in repair costs due to earthquake damage has been proposed. This procedure works within the framework proposed by PEER for performance-based earthquake engineering. Total cost defined is a function of repair costs for individual building elements, except in the collapse case, where a separate cost estimation is used. Identified aleatory and epistemic uncertainty in ground motion hazard, building response, damage to building elements and element repair costs is combined to produce an uncertainty in total repair cost. The proposed procedure uses the First-Order Second-Moment (FOSM) method to collapse several conditional random variables into a single random variable. Numerical integration is then used to incorporate the ground motion hazard, where the uncertainty is most significant. The resulting information is expected annual loss, variance in annual loss, and the annual rate of exceeding a given cost.

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